Analyzing Angles

We know what angles 1 and 2 are in terms of x, so we need to figure out how they are related to each other. One thing to note in the picture is that lines AB and CE are parallel since they are both perpendicular to line AC. We can see from the picture that $\angle 1$ is congruent to $\angle DBA$ because they are alternate interior angles (with BC as the transversal).

Now we can see that $\angle 2$ is supplementary to $\angle DBA$. So, $\angle 2$ must be supplementary to $\angle 1$ as well (since 1 and DBA are congruent). This means we can make the following equation, which we can solve for x:

$m\angle 1 + m\angle 2 = 180°$

$(2x + 10) + (5x - 40) = 180$

$7x - 30 = 180$

$7x = 210$ so $x = 30$

We can use this to find the measures of angles 1 and 2.

$m\angle 1 = 2x + 10$ $\qquad$ $m\angle 2 = 5x - 40$

$m\angle 1 = 2(30) + 10$ $\qquad$ $m\angle 2 = 5(30) - 40$

$m\angle 1 = 70°$ $\qquad$ $m\angle 2 = 110°$

We know that this is correct since 70 + 110 = 180, which is as it should be.

Now we need to find the measure of angle 3. We can see that $\angle 3 \cong \angle DAB$, since they're vertical angles. We know the measures of angles DBA and BDA, and we know that the sum of the measure of the angles in a triangle is 180. So, we can write the following equation, which we can solve for $\angle DAB$.

$m\angle DBA + m\angle BDA + m\angle DAB = 180°$

$70 + 60 + m\angle DAB = 180$

$130 + m\angle DAB = 180$

$m\angle DAB = 50°$ so $m\angle 3 = 50°$

Extra: If we don't know the measure of angle 4, we can't find the angle measure of angle 3. But, we can say some things about its angle measure.

We assume that E is on the same side of CA as B is, and that it's still on the purple line (from C). First, angle 3 must be between 0 and 90. We also know that $m\angle 2 = m\angle 4 + m\angle 3$ (actually the vertical angle to 3, but we'll just call it 3), since the exterior angle of a triangle equals the sum of a remote interior angles. Therefore, $m\angle 3 + m\angle 4 = 100°$. Beyond that, we can't say much.

